1. Of all the solids of the type shown above where the volume is 2025 cubic centimeters, find the values of x and h which minimize the surface area. Note that the surface area consists of the areas of the top and bottom I-shapes plus the areas of the twelve rectangles that form the sides. You must provide analysis that a particular critical number does indeed produce the minimum surface area.  

2. a) Give formulas for the two functions y that are implicitly defined by the relation: 
   \[(x^2 + 1)y^2 + 6y + 1 = 0\]  
   (2) 
   
   b) Use implicit differentiation to solve for \(\frac{dy}{dx}\) in terms of x and y and then find the slope-intercept equation of the line that is tangent to the graph of the relation at the point (2,-1).  
   (4) 
   c) In the window below, accurately draw the graphs of the relation 
   \[(x^2 + 1)y^2 + 6y + 1 = 0\] and the tangent line at the point (2,-1).  
   (3)
3. Give the formula for a function that has a graph similar to the one above. The asymptotes of this graph are the lines $x = -2$, $x = 1$ and $y = 1$. The $y$-intercept is $(0,1)$. You must explain why the graph of your function has these features. (3)

4. a) Find the slope-intercept equation of the tangent line to the curve $y = x^4 - 6x^3 + 9x^2 + x$ at the point $(1,5)$. (3)
   
   b) Prove that the line $y = x$ is tangent to the curve $y = x^4 - 6x^3 + 9x^2 + x$ at two points. (3)

5. Calculate the following limits. You may not use L'Hospital's Rule. You must show all your work. (3 marks each)
   
   a) $\lim_{x \to 5} \frac{x^3 - 125}{x^7 - x - 20}$
   
   b) $\lim_{x \to 3} \frac{2x - 10}{3 - \sqrt{4}}$
   
   c) $\lim_{x \to 0} \frac{x \sin(x)}{1 - \cos(x)}$
   
   d) $\lim_{x \to \frac{3}{2n}} \left(1 + \frac{3}{2n}\right)^{4n}$

6. Use logarithmic differentiation to find the derivative of $f(x) = \frac{\sin^3 x + xe^{3x}}{\sqrt{8x^2 + 1}}$. (4)

7. Using only the definition of $f'(x)$, find the derivative of $f(x) = 2x^3 - 4x^2 + 3x + 6$. (4)

8. Let $f(x) = \frac{x}{x^3 + 3}$. Find all the intercepts, asymptotes, relative extrema and inflection points of the graph of $f(x)$. You must include the proper analysis using the first and second derivatives of $f$. (8)
9. a) The exponential growth (decay) model described by a function \( A(t) \) whose rate of change is proportional to

\[
\text{and is a solution of the differential equation } A'(t) = \ldots \text{ and is of the form } A(t) = \ldots \quad (3)
\]

b) The National Gallery of Post-Modern Canadian Art recently discovered that one of its most prized possessions, an oil painting entitled *Fibonacci Rabbits*, is a forgery. It is well known that the great Canadian artist, Lanny Semenko, painted his masterpiece, *Fibonacci Rabbits*, in the summer of 1980. However, an analysis of a paint sample taken from the forged painting at the National Gallery reveals that 99.871% of its original C14 is still present which means that this painting was painted well after 1980. Given that the half-life of C14 is 5730 years, what is the earliest date that the forgery could have been painted? (3)

10. Let \( f(x) = x^{2/3} + \frac{2}{3}x \). Find all the critical numbers of \( f(x) \) on \((-2, 1)\) and then find the absolute extrema of \( f(x) \) on \([-2, 1]\). (4)

11. a) The Logistic Growth Model is a function \( Q(t) \) that has a maximum value \( M \) and satisfies the differential equation \( Q'(t) = \ldots \). All solutions of this equation are of the form \( Q(t) = \ldots \quad (2) \)

b) Tiny Ted, shown above, grows according to the logistic growth model. At birth, he weighed 1 pound and at six months, he weighed 6 pounds. Assuming that he will never weigh more than 9 pounds, what will he weigh when he is 9 months old? (3)
12. Suppose that the function \( f(x) \) is defined by two formulas: when \( x \leq 2 \), \( f(x) = 2x \) and when \( x > 2 \) then \( f(x) = x^2 + x - 2 \). Prove that \( f \) is continuous at \( x = 2 \) but is not differentiable at \( x = 2 \). \( \text{(5)} \)

13. a) Accurately draw the graphs of \( y = e^x \) and \( y = 2\cos(x) \) in the window \( 0 \leq x \leq \pi/2, \ 0 < y < 5 \) and then zoom to find the x-coordinate of the intersection point of these graphs rounded to one decimal place. \( \text{(3)} \)
   
   b) Using the value found in part a) as \( x_n \), use Newton’s method to find the positive zero of \( f(x) = e^x - 2\cos(x) \) rounded to six decimal places. Gives the values of the iterations rounded to six decimal places and also give the formula used to calculate the iterations. \( \text{(3)} \)

14. Accurately draw the graph of a function \( F(x) \) that has all the following properties:
   
   - \( F \) is continuous on \((0,6)\) and \((6,8)\)
   - \( F(0) = 1 \) and \( F'(x) = \frac{1}{2} \) on \((0,2)\)
   - \( F(4) = 0 \) and on \((2,4)\) \( F'(x) < 0 \) and \( F''(x) > 0 \)
   - On \((4,6)\), \( F''(x) < 0 \) and \( \lim_{x \to 0} F(x) = -\infty \)
   - On \((6,8)\), \( F''(x) > 0 \) and \( \lim_{x \to 0} F(x) = \infty \) and \( F(8) = 0 \) \( \text{(4)} \)

15. A manufacturer estimates that \( x(p) = 5000 - \frac{1}{200} p^3 \) units of a particular commodity will be sold when the price is \( p \) dollars per unit, for \( 0 \leq p \leq 100 \)
   
   a) Express the elasticity of demand as a function of \( p \) \( \text{(2)} \)
   
   b) Find all the values of \( p \) where the demand is elastic. \( \text{(2)} \)
   
   c) Find the maximum possible revenue. Give supporting analysis. \( \text{(3)} \)

16. Find the particular solution of the differential equation \( \frac{dy}{dx} = y^3 \sqrt[4]{6-x} \) that the satisfies the condition that \( y = 1 \) when \( x = 4 \). \( \text{(5)} \)

17. Solve the following indefinite integrals. Show all work. Express your solutions in terms of the original variable \( x \).
   
   a) \( \int \frac{2x^3 + 3x^2}{\sqrt{x^4 + 2x^3 + 1}} \, dx \) \( \text{(3)} \)
   
   b) \( \int \frac{3}{x(\ln x)^4} \, dx \) \( \text{(3)} \)

Total = 100